AN INTEGRATED LEARNING SYSTEM

# MECHANICS OF MATERIALS

Timothy A. Philpot

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# MECHANICS OF MATERIALS:
# An Integrated Learning System

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# MECHANICS OF MATERIALS: An Integrated Learning System

**Timothy A. Philpot**
*Missouri University of Science and Technology*
*Rolla, Missouri*

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**Library of Congress Cataloging-in-Publication Data**

Philpot, Timothy A.
Mechanics of materials : an integrated learning system / Timothy A. Philpot. — 3rd ed.
p. cm.
ISBN 978-1-118-08347-5 (hardback)
1. Materials—Mechanical properties. 2. Strength of materials. I. Title.

TA405.P4884 2012
620.1'123—dc23

2012017380

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

# About the Author

**Timothy A. Philpot** is an Associate Professor in the Department of Civil, Architectural, and Environmental Engineering at the Missouri University of Science and Technology (formerly known as the University of Missouri–Rolla). He received his B.S. degree from the University of Kentucky in 1979, his M.Engr. degree from Cornell University in 1980, and his Ph.D. degree from Purdue University in 1992. In the 1980s, he worked as a structural engineer in the offshore construction industry in New Orleans, London, Houston, and Singapore. He joined the faculty at Murray State University in 1986, and since 1999, he has been on the faculty at Missouri S & T.

Dr. Philpot's primary areas of teaching and research are in engineering mechanics and the development of interactive, multimedia educational software for the introductory engineering mechanics courses. He is the developer of *MDSolids* and *MecMovies*, two award-winning instructional software packages. *MDSolids–Educational Software for Mechanics of Materials* won a 1998 Premier Award for Excellence in Engineering Education Courseware by NEEDS, the National Engineering Education Delivery System. *MecMovies* was a winner of the 2004 NEEDS Premier Award competition as well as a winner of the 2006 MERLOT Classics and MERLOT Editors' Choice Awards for Exemplary Online Learning Resources. Dr. Philpot is also a certified *Project Lead the Way* affiliate professor for the Principles of Engineering course, which features *MDSolids* in the curriculum.

He is a licensed professional engineer and a member of the American Society of Civil Engineers and the American Society for Engineering Education. He has been active in leadership of the ASEE Mechanics Division.

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# Preface

At the beginning of each semester, I always tell my students the story of my undergraduate Mechanics of Materials experience. While I somehow managed to make an A in the course, Mechanics of Materials was one of the most confusing courses in my undergraduate curriculum. As I continued my studies, I found that I really didn't understand the course concepts well, and this weakness hindered my understanding of subsequent design courses. It wasn't until I began my career as an engineer that I began to relate the Mechanics of Materials concepts to specific design situations. Once I made that real-world connection, I understood the design procedures associated with my discipline more completely and I developed confidence as a designer. My educational and work-related experiences convinced me of the central importance of the Mechanics of Materials course as the foundation for advanced design courses and engineering practice.

## The Education of the Mind's Eye

As I gained experience during my early teaching career, it occurred to me that I was able to understand and explain the Mechanics of Materials concepts because I relied upon a set of mental images that facilitated my understanding of the subject. Years later, during a formative assessment of the MecMovies software, Dr. Andrew Dillon, Dean of the School of Information at the University of Texas at Austin, succinctly expressed the role of mental imagery in the following way: "A defining characteristic of an expert is that an expert has a strong mental image of his or her area of expertise while a novice does not." Based on this insight, it seemed logical that one of the instructor's primary objectives should be to teach to the mind's eye—conveying and cultivating relevant mental images that inform and guide students in the study of Mechanics of Materials. The illustrations as well as the MecMovies software integrated in this book have been developed with this objective in mind.

## MecMovies Instructional Software

Computer-based instruction often enhances the student's understanding of Mechanics of Materials. With three-dimensional modeling and rendering software, it is possible to create photo-realistic images of various components and to show these components from various viewpoints. In addition, animation software allows objects or processes to be shown in motion. By combining these two capabilities, a fuller description of a physical object can be presented, which can facilitate the mental visualization so integral to understanding and solving engineering problems.

Animation also offers a new generation of computer-based learning tools. The traditional instructional means used to teach Mechanics of Materials—example problems—can be greatly enhanced through animation by emphasizing and illustrating desired problem-solving processes in a more memorable and engaging way. Animation can be used to create interactive tools that focus on specific skills students need to become proficient problem-solvers. These computer-based tools can provide not only the correct solution, but also a detailed visual and verbal explanation of the process needed to arrive at the solution. The feedback provided by the software can lessen some of the anxiety typically associated with traditional homework assignments, while also enabling learners to build their competence and confidence at a pace that is right for them.

This book integrates computer-based instruction into the traditional textbook format with the addition of the MecMovies instructional software. At present, MecMovies consists of over 160 animated "movies" on topics spanning the breadth of the Mechanics of Materials course. Most of these animations present detailed example problems, and about 80 movies are interactive, providing learners with the opportunity to apply concepts and receive immediate feedback that includes key considerations, calculation details, and intermediate results. MecMovies was a winner of the 2004 Premier Award for Excellence in Engineering Education Courseware presented by NEEDS (the National Engineering Education Delivery System, a digital library of learning resources for engineering education).

## Hallmarks of the Textbook

In 26 years of teaching the fundamental topics of strength, deformation, and stability, I have encountered successes and frustrations, and I have learned from both. This book has grown out of a passion for clear communication between instructor and student and a drive for documented effectiveness in conveying this foundational material to the differing learners in my classes. With this book and the MecMovies instructional software that is integrated throughout, my desire is to present and develop the theory and practice of Mechanics of Materials in a straightforward plain-speaking manner that addresses the needs of varied learners. The text and software strive to be "student-friendly" without sacrificing rigor or depth in the presentation of topics.

**Communicating visually:** I invite you to thumb through this book. My hope is that you will find a refreshing clarity in both the text and the illustrations. As both the author and the illustrator, I've tried to produce visual content that will help illuminate the subject matter for the mind's eye of the reader. The illustrations use color, shading, perspective, texture, and dimension to convey concepts clearly, while aiming to place these concepts in the context of real-world components and objects. These illustrations have been prepared by an engineer to be used by engineers to train future engineers.

**Problem-solving schema:** Educational research suggests that transfer of learning is more effective when students are able to develop *problem-solving schema*, which Webster's Dictionary defines as "a mental codification that includes an organized way of responding to a complex situation." In other words, understanding and proficiency are enhanced if students are encouraged to build a structured framework for mentally organizing concepts and their method of application. This book and software include a number of features aimed at helping students to organize and categorize the Mechanics of Materials concepts and problem-solving procedures. For instance, experience has shown that statically indeterminate axial and torsion structures are among the most difficult topics for students. To help organize the solution process for these topics, a five-step method is utilized. This approach provides students with a problem-solving method that systematically transforms a potentially confusing situation into an easily understandable calculation procedure. Summary tables are also presented in these topics to help students place common statically indeterminate structures into categories based on the specific geometry of the structure. Another topic that students typically find confusing is the use of the superposition method to determine beam deflections. This topic is introduced in the text through enumeration of eight simple skills commonly used in solving problems of this type. This organizational scheme allows students to develop proficiency incrementally before considering more complex configurations.

**Style and clarity of examples:** To a great extent, the Mechanics of Materials course is taught through examples, and consequently, this book places great emphasis on the presentation and quality of example problems. The commentary and the illustrations associated with example problems are particularly important to the learner. The commentary explains why various steps are taken and describes the rationale for each step in the solution process, while the illustrations help build the mental imagery needed to transfer the concepts to differing situations. Students have found the step-by-step approach used in MecMovies to be particularly helpful, and a similar style is used in the text. Altogether, this book and the MecMovies software present more than 270 fully illustrated example problems that provide both the breadth and the depth required to develop competency and confidence in problem-solving skills.

**Homework philosophy:** Since Mechanics of Materials is a problem-solving course, much deliberation has gone into the development of homework problems that elucidate and reinforce the course concepts. This book includes 1200 homework problems in a range of difficulty suitable for learners at various stages of development. These problems have been designed with the intent of building the technical foundation and skills that will be necessary in subsequent engineering design courses. The problems are intended to be challenging, and at the same time, practical and pertinent to traditional engineering practice.

## New in the Third Edition

- Two new sections have been added in Chapter 9 to discuss additional topics related to shear stress in beams:
  - **9.9 Shear Stress and Shear Flow in Thin-Walled Members**
  - **9.10 Shear Centers of Thin-Walled Open Sections**
- Chapter 17, “Energy Methods,” has been developed to discuss the application of work and strain energy principles, virtual work principles, and Castigliano’s Theorem to solid mechanics problems.
- Design equations in Chapter 16 for the critical buckling stress of structural steel columns have been updated to conform to the latest provisions of ANSI/AISC 360-10 *Specification for Structural Steel Buildings.*
- A number of changes have been made to the textbook problems: Of the problems that appeared in the second edition, 190 have been revised (16 percent of all the problems in the book), and 300 new problems have been added (25 percent). About half of the added problems are associated with the new material in Chapters 9 and 17. The other 150 problems have been added to broaden the variety of problems available for many topics.

# Incorporating MecMovies into Course Assignments

Some instructors may have had unsatisfying experiences with instructional software in the past. Often, the results have not matched the expectations, and it is understandable that instructors may be reluctant to incorporate computer-based instructional content into their course. For those instructors, this book can stand completely on its own merits without the need for the MecMovies software. Instructors will find that this book can be used to successfully teach the time-honored Mechanics of Materials course without making use of the MecMovies software in any way. However, the MecMovies software integrated into this book is a new and valuable instructional medium that has proven to be both popular and effective with Mechanics of Materials students. Naysayers may argue that for many years instructional software has been included as supplemental material in textbooks, and it has not produced significant changes in student performance. While I cannot disagree with this assessment, let me try to persuade you to view MecMovies differently.

Experience has shown that the *manner* in which instructional software is integrated into a course is just as important as the quality of the software itself. Students have many demands on their study time, and in general, they will not invest their time and effort in software that they perceive to be peripheral to the course requirements. In other words, *supplementary* software is doomed to failure, regardless of its quality or merit. To be effective, instructional software must be *integrated into the course assignments* on a regular and frequent basis. Why would you as an instructor alter your traditional teaching routine to integrate computer-based assignments into your course? The answer is because the unique capabilities offered by MecMovies can (a) provide individualized instruction to your students, (b) enable you to spend more time discussing advanced rather than introductory aspects of many topics, and (c) make your teaching efforts more effective.

The computer as an instructional medium is well suited for individualized interactive learning exercises, particularly for those skills that require repetition to master. MecMovies has many interactive exercises, and at a minimum, these features can be utilized by instructors to (a) ensure that students have the appropriate skills in prerequisite topics such as centroids and moments of inertia, (b) develop necessary proficiency in specific problem-solving skills, and (c) encourage students to stay up to date with lecture topics. Three types of interactive features are included in MecMovies:

1. **Concept Checkpoints** – This feature is used for rudimentary problems requiring only one or two calculations. It is also used to build proficiency and confidence in more complicated problems by subdividing the solution process into a sequence of steps that can be mastered sequentially.
2. **Try One problems** – This feature is appended to specific example problems. In a Try One problem, the student is presented with a problem similar to the example so that he or she has the opportunity to immediately apply the concepts and problem-solving procedures illustrated in the example.
3. **Games** – Games are used to develop proficiency in specific skills that require repetition to master. For example, games are used to teach centroids, moments of inertia, shear-force and bending-moment diagrams, and Mohr's circle.

With each of these software features, numeric values in the problem statement are dynamically generated for each student, the student's answers are evaluated, and a summary report suitable for printing is generated. *This enables daily assignments to be collected without imposing a grading burden on the instructor*.

Many of the interactive MecMovies exercises assume no prior knowledge of the topic. Consequently, an instructor can require a *MecMovies* feature to be completed *before giving a lecture on the topic*. For example, Coach Mohr's Circle of Stress guides students step by step through the details of constructing Mohr's circle for plane stress. If students complete this exercise before attending the first Mohr's circle lecture, then the instructor can be confident that students will have at least a basic understanding of how to use Mohr's circle to determine principal stresses. The instructor is then free to build upon this basic level of understanding to explain additional aspects of Mohr's circle calculations.

Student response to MecMovies has been excellent. Many students report that they prefer studying from MecMovies rather than from the text. Students quickly find that MecMovies does indeed help them understand the course material better and thus score better on exams. Furthermore, less quantifiable benefits have been observed when MecMovies is integrated into the course. Students are able to ask better, more specific questions in class concerning aspects of theory that they don't yet fully understand, and students' attitudes about the course overall seem to improve.

## Acknowledgements

- Thanks to Linda Ratts, Chris Teja, and the hard working, efficient, and focused staff at Wiley for keeping this project on track.
- Thanks to Dr. Jeffrey S. Thomas at Missouri S&T for his excellent work on the WileyPLUS content for this textbook. Thanks for your innovation, dedication, and commitment to this effort.
- Thanks to Jackie Henry at Aptara, Inc., and to Ellen Sanders and Brian Baker of Write With, Inc., for their great work in editing and preparing the manuscript for production.
- And finally to Pooch, my wife, the mother of my children, the love of my life, and my constant companion for the past 40 years. Words are inadequate to convey the depth of love, support, strength, encouragement, optimism, wisdom, enthusiasm, good humor, and sustenance that you give so freely to me.

The following colleagues in the engineering teaching profession reviewed parts or all of the manuscript, and I am deeply indebted to them for their constructive criticism and words of encouragement.

**Second Edition.** John Baker, *University of Kentucky*; George R. Buchanan, *Tennessee Technological University*; Debra Dudick, *Corning Community College*; Yalcin Ertekin, *Trine University*; Nicholas Xuanlai Fang, *University of Illinois Urbana-Champaign*; Noe Vargas Hernandez, *University of Texas at El Paso*; Ernst W. Kiesling, *Texas Tech University*; Lee L. Lowery, Jr., *Texas A&M University*; Kenneth S. Manning, *Adirondack Community College*; Prasad S. Mokashi, *Ohio State University*; Ardavan Motahari, *University of Texas at Arlington*; Dustyn Roberts, *New York University*; Zhong-Sheng Wang, *Embry-Riddle Aeronautical University*.

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## Contact Information

I would greatly appreciate your comments and suggestions concerning this book and the MecMovies software. Please feel free to send me an e-mail message at *philpott@mst.edu* or *philpott@mdsolids.com*.

# Contents

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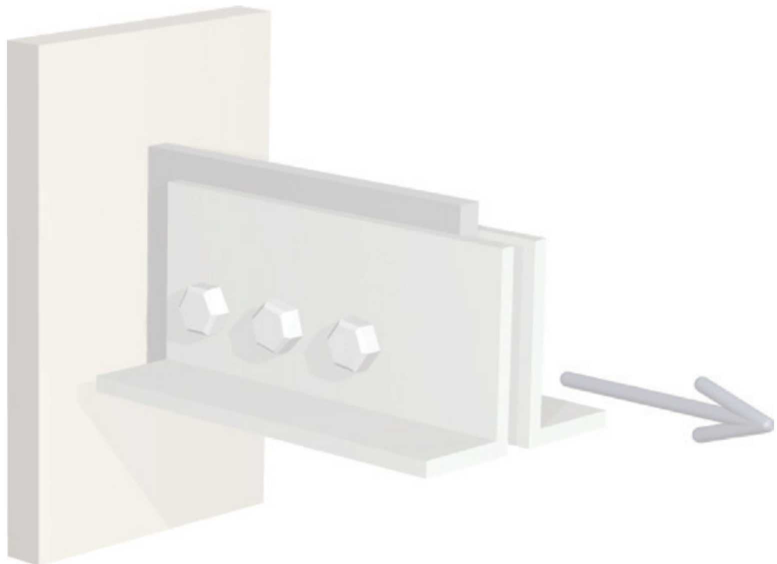
CHAPTER 1

# Stress



## 1.1 Introduction

The three fundamental areas of engineering mechanics are statics, dynamics, and mechanics of materials. Statics and dynamics are devoted primarily to the study of *external* forces and motions associated with particles and rigid bodies (i.e., idealized objects in which any change of size or shape due to forces can be neglected). Mechanics of materials is the study of the *internal* effects caused by external loads acting on real bodies that deform (meaning objects that can stretch, bend, or twist). Why are the internal effects in an object important? Engineers are called upon to design and produce a variety of objects and structures such as automobiles, airplanes, ships, pipelines, bridges, buildings, tunnels, retaining walls, motors, and machines. Regardless of the application, however, a safe and successful design must address the following three mechanical concerns:

1. **Strength:** Is the object strong enough to withstand the loads that will be applied to it? Will it break or fracture? Will it continue to perform properly under repeated loadings?
2. **Stiffness:** Will the object deflect or deform so much that it cannot perform its intended function?
3. **Stability:** Will the object suddenly bend or buckle out of shape at some elevated load so that it can no longer continue to perform its function?

Addressing these concerns requires both an assessment of the intensity of internal forces and deformations acting within the body and an understanding of the mechanical characteristics of the material used to make the object.

Mechanics of materials is a basic subject in many engineering fields. The course focuses on several types of components: bars subjected to axial loads, shafts in torsion, beams in bending, and columns in compression. Numerous formulas and rules for design found in engineering codes and specifications are based on mechanics of materials fundamentals associated with these types of components. With a strong foundation in mechanics of materials concepts and problem-solving skills, the student is well equipped to continue into more advanced engineering design courses.

## 1.2 Normal Stress Under Axial Loading

In every subject area, there are certain fundamental concepts that assume paramount importance for a satisfactory comprehension of the subject matter. In mechanics of materials, such a concept is that of **stress**. In the simplest qualitative terms, *stress is the intensity of internal force*. Force is a vector quantity and as such has both magnitude and direction. Intensity implies an area over which the force is distributed. Therefore, stress can be defined as

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \tag{1.1}$$

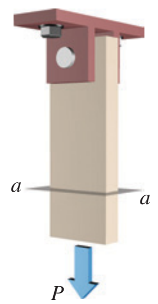



**FIGURE 1.1*a*** Bar with axial load *P*.

To introduce the concept of a **normal stress**, consider a rectangular bar subjected to an axial force (Figure 1.1*a*). An **axial force** is a load that is directed along the longitudinal axis of the member. Axial forces that tend to elongate a member are termed **tension forces**, and forces that tend to shorten a member are termed **compression forces**. The axial force *P* in Figure 1.1*a* is a tension force. To investigate internal effects, the bar is cut by a transverse plane, such as plane *a–a* of Figure 1.1*a*, to expose a free-body diagram of the bottom half of the bar (Figure 1.1*b*). Since this cutting plane is perpendicular to the longitudinal axis of the bar, the exposed surface is called a **cross section**.

> The technique of cutting an object to expose the internal forces acting on a plane surface is often referred to as the **method of sections**. The cutting plane is called the **section plane**. To investigate internal effects, one might simply say something like "Cut a section through the bar" to imply the use of the method of sections technique. This technique will be used throughout the study of mechanics of materials to investigate the internal effects caused by external forces acting on a solid body.

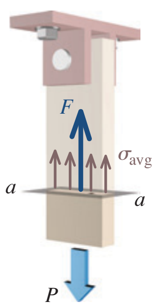



**FIGURE 1.1*b*** Average stress.

Equilibrium of the lower portion of the bar is attained by a distribution of internal force that develops on the exposed cross section. This distribution of internal force has a resultant *F* that is normal to the exposed surface, is equal in magnitude to *P*, and has a line of action that is collinear with the line of action of *P*. The intensity of distributed internal force acting in the material is referred to as stress.

In this instance, the stress acts on a surface that is *perpendicular* to the direction of the internal force. A stress of this type is called a **normal stress**, and it is denoted by the Greek letter $\sigma$ (sigma). To determine the magnitude of the normal stress in the bar, the average intensity of internal force on the cross section can be computed as

$$\sigma_{\text{avg}} = \frac{F}{A} \tag{1.2}$$

where $A$ is the cross-sectional area of the bar.

The **sign convention** for normal stresses is defined as follows:

- A positive sign indicates a *tension normal stress*, and
- a negative sign denotes a *compression normal stress*.

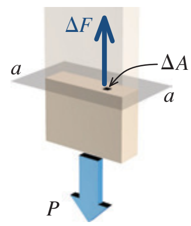


**FIGURE 1.1c** Stress at a point.

Consider now a small area $\Delta A$ on the exposed cross section of the bar, as shown in Figure 1.1$c$, and let $\Delta F$ represent the resultant of the internal forces transmitted in this small area. The average intensity of the internal force being transmitted in area $\Delta A$ is obtained by dividing $\Delta F$ by $\Delta A$. If the internal forces transmitted across the section are assumed to be uniformly distributed, the area $\Delta A$ can be made smaller and smaller, and in the limit, it will approach a point on the exposed surface. The corresponding force $\Delta F$ also becomes smaller and smaller. The stress at the point on the cross section to which $\Delta A$ converges is defined as

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.3}$$

If the distribution of stress is to be uniform, as in Equation (1.2), the resultant force must act through the centroid of the cross-sectional area. For long, slender, axially loaded members, such as those found in trusses and similar structures, it is generally assumed that the normal stress is uniformly distributed except near the points where external load is applied. Stress distributions in axially loaded members are not uniform near holes, grooves, fillets, and other features. These situations will be discussed in later sections on stress concentrations. *In this book, it is understood that axial forces are applied at the centroids of the cross sections unless specifically stated otherwise.*

## Stress Units

Since the normal stress is computed by dividing the internal force by the cross-sectional area, stress has the dimensions of force per unit area. When U.S. Customary units are used, stress is commonly expressed in pounds per square inch (psi) or kips per square inch (ksi) where 1 kip = 1,000 lb. When the International System of Units, universally abbreviated SI (from the French *Le Système International d'Unités*), is used, stress is expressed in pascals (Pa) and computed as force in newtons (N) divided by area in square meters ($m^2$). For typical engineering applications, the pascal is a very small unit and, therefore, stress is more commonly expressed in megapascals (MPa) where 1 MPa = 1,000,000 Pa. A convenient alternative when calculating stress in MPa is to express force in newtons and area in square millimeters ($mm^2$). Therefore,

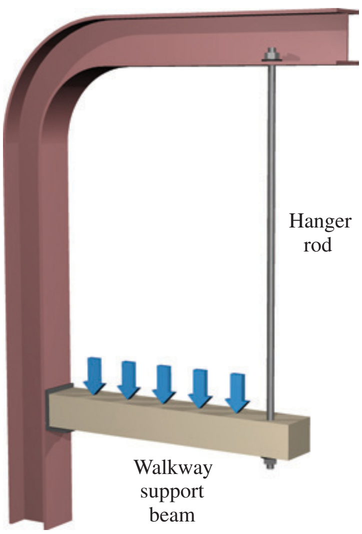
$$1 \text{ MPa} = 1{,}000{,}000 \text{ N/m}^2 = 1 \text{ N/mm}^2 \tag{1.4}$$

## Significant Digits

In this book, final numerical answers are usually presented with three significant digits when a number begins with the digits 2 through 9, and with four significant digits when the number begins with the digit 1. Intermediate values are generally recorded with additional digits to minimize the loss of numerical accuracy due to rounding.

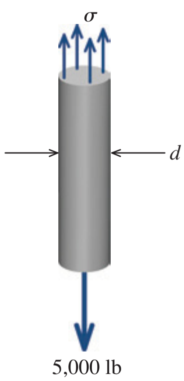
In developing stress concepts through example problems and exercises, it is convenient to use the notion of a **rigid element**. Depending on how it is supported, a rigid element may move vertically or horizontally, or it may rotate about a support location. The rigid element is assumed to be infinitely strong.

## EXAMPLE 1.1




A solid 0.5-in.-diameter steel hanger rod is used to hold up one end of a walkway support beam. The force carried by the rod is 5,000 lb. Determine the normal stress in the rod. (Disregard the weight of the rod.)

### SOLUTION




**Free-body diagram of hanger rod.**

A free-body diagram of the rod is shown. The solid rod has a circular cross section, and its area is computed as

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.5\text{ in.})^2 = 0.19635\text{ in.}^2$$

where $d$ = rod diameter.

Since the force in the rod is 5,000 lb, the normal stress in the rod can be computed as

$$\sigma = \frac{F}{A} = \frac{5{,}000\text{ lb}}{0.19635\text{ in.}^2} = 25{,}464.73135\text{ psi}$$

Although this answer is numerically correct, it would not be proper to report a stress of 25,464.73135 psi as the final answer. A number with this many digits implies an accuracy that we have no right to claim. In this instance, both the rod diameter and the force are given with only one significant digit of accuracy; however, the stress value we have computed here has 10 significant digits.

In engineering, it is customary to round the final answers to three significant digits (if the first digit is not 1) or four significant digits (if the first digit is 1). Using this guideline, the normal stress in the rod would be reported as

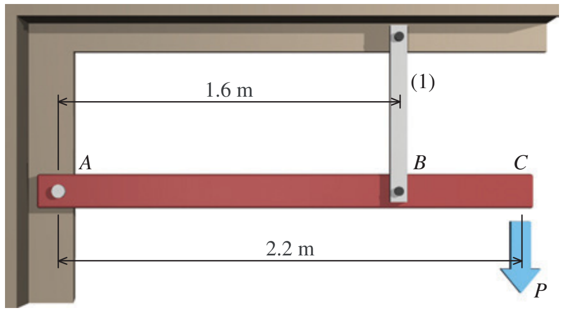
$$\sigma = 25{,}500\text{ psi}$$ **Ans.**

In many instances, the illustrations in this book attempt to show objects in realistic three-dimensional perspective. Wherever possible, an effort has been made to show free-body diagrams within the actual context of the object or structure. In these illustrations, the free-body diagram is shown in full color, while other portions of the object or structure are faded out.

## EXAMPLE 1.2

Rigid bar *ABC* is supported by a pin at *A* and axial member (1), which has a cross-sectional area of 540 mm². The weight of rigid bar *ABC* can be neglected. (**Note:** 1 kN = 1,000 N.)

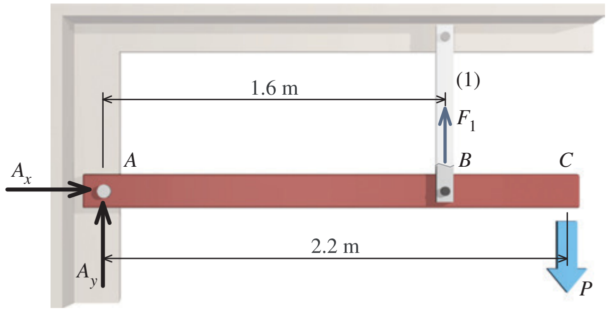
(a) Determine the normal stress in member (1) if a load of $P = 8$ kN is applied at *C*.
(b) If the maximum normal stress in member (1) must be limited to 50 MPa, what is the maximum load magnitude *P* that may be applied to the rigid bar at *C*?




### Plan the Solution

**(Part a)**

Before the normal stress in member (1) can be computed, its axial force must be determined. To compute this force, consider a free-body diagram of rigid bar *ABC* and write a moment equilibrium equation about pin *A*.




**Free-body diagram of rigid bar *ABC*.**

### SOLUTION

**(Part a)**

For rigid bar *ABC*, write the equilibrium equation for the sum of moments about pin *A*. Let $F_1$ = internal force in member (1) and assume that $F_1$ is a tension force. Positive moments in the equilibrium equation are defined by the right-hand rule.

$$\Sigma M_A = -(8 \text{ kN})(2.2 \text{ m}) + (1.6 \text{ m})F_1 = 0$$
$$\therefore F_1 = 11 \text{ kN}$$

The normal stress in member (1) can be computed as

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(11 \text{ kN})(1{,}000 \text{ N/kN})}{540 \text{ mm}^2} = 20.370 \text{ N/mm}^2 = 20.4 \text{ MPa} \qquad \textbf{Ans.}$$

(Note the use of the conversion factor 1 MPa = 1 N/mm².)

### Plan the Solution

**(Part b)**

Using the stress given, compute the maximum force that member (1) may safely carry. Once this force is computed, use the moment equilibrium equation to determine the load *P*.

### SOLUTION

**(Part b)**

Determine the maximum force allowed for member (1):

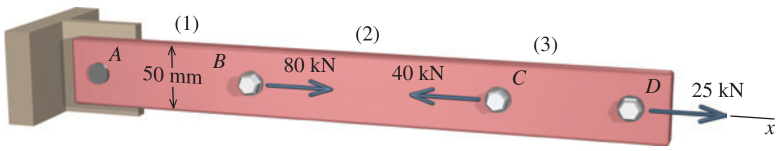
$$\sigma = \frac{F}{A}$$

$$\therefore F_1 = \sigma_1 A_1 = (50 \text{ MPa})(540 \text{ mm}^2) = (50 \text{ N/mm}^2)(540 \text{ mm}^2) = 27{,}000 \text{ N} = 27 \text{ kN}$$

Compute the maximum allowable load *P* from the moment equilibrium equation:

$$\Sigma M_A = -(2.2 \text{ m})P + (1.6 \text{ m})(27 \text{ kN}) = 0$$
$$\therefore P = 19.64 \text{ kN} \qquad \textbf{Ans.}$$

## EXAMPLE 1.3




A 50-mm-wide steel bar has axial loads applied at points *B*, *C*, and *D*. If the normal stress magnitude in the bar must not exceed 60 MPa, determine the minimum thickness that can be used for the bar.

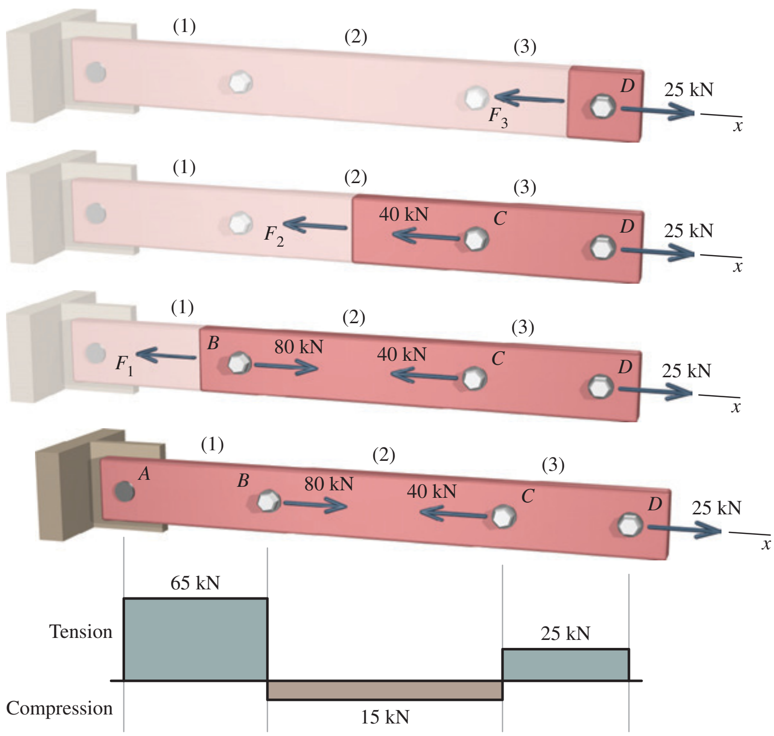
### Plan the Solution

Draw free-body diagrams that expose the internal force in each of the three segments. Determine the magnitude and direction of the internal axial force in each segment required to satisfy equilibrium. Use the largest internal axial force magnitude and the allowable normal stress to compute the minimum cross-sectional area required for the bar. Divide the cross-sectional area by the 50-mm bar width to compute the minimum bar thickness.

### SOLUTION

Begin by drawing a free-body diagram (FBD) that exposes the internal force in segment (3). Since the reaction force at *A* has not been calculated, it will be easier to cut through the bar in segment (3) and consider the portion of the bar starting at the cut surface and extending to the free end of the bar at *D*. An unknown internal axial force $F_3$ exists in segment (3), and it is helpful to establish a consistent convention for problems of this type.

> **Problem-Solving Tip:** When cutting a FBD through an axial member, assume that the internal force is tension and draw the force arrow directed *away from the cut surface*. If the computed internal force value turns out to be a positive number, then the assumption of tension is confirmed. If the computed value turns out to be a negative number, then the internal force is actually compression.




**Axial-force diagram showing internal forces in each bar segment.**

Based on a FBD cut through axial segment (3), the equilibrium equation is

$$\Sigma F_x = -F_3 + 25 \text{ kN} = 0$$

$$\therefore F_3 = 25 \text{ kN} = 25 \text{ kN (T)}$$

Repeat this procedure for a FBD exposing the internal force in segment (2),

$$\Sigma F_x = -F_2 - 40 \text{ kN} + 25 \text{ kN} = 0$$

$$\therefore F_2 = -15 \text{ kN} = 15 \text{ kN (C)},$$

and for a FBD exposing the internal force in segment (1),

$$\Sigma F_x = -F_1 + 80 \text{ kN} - 40 \text{ kN} + 25 \text{ kN} = 0$$

$$\therefore F_1 = 65 \text{ kN (T)}$$

It is always a good practice to construct a simple plot that graphically summarizes the internal axial forces along the bar. The axial-force diagram on the left shows internal tension forces above the axis and internal compression forces below the axis.

The required cross-sectional area will be computed on the basis of the largest internal force magnitude (i.e., absolute value). The normal stress in the bar must be limited to 60 MPa. *To facilitate the calculation, the conversion 1 MPa = 1 N/mm² is used; therefore, 60 MPa = 60 N/mm².*

$$\sigma = \frac{F}{A} \qquad \therefore A \geq \frac{F}{\sigma} = \frac{(65 \text{ kN})(1{,}000 \text{ N/kN})}{60 \text{ N/mm}^2} = 1{,}083.333 \text{ mm}^2$$

Since the flat steel bar is 50 mm wide, the minimum thickness that can be used for the bar is

$$t_{\text{min}} \geq \frac{1{,}083{,}333 \text{ mm}^2}{50 \text{ mm}} = 21.667 \text{ mm} = 21.7 \text{ mm} \qquad \textbf{Ans.}$$

In practice, the bar thickness would be rounded up to the next larger standard size.

**Review**

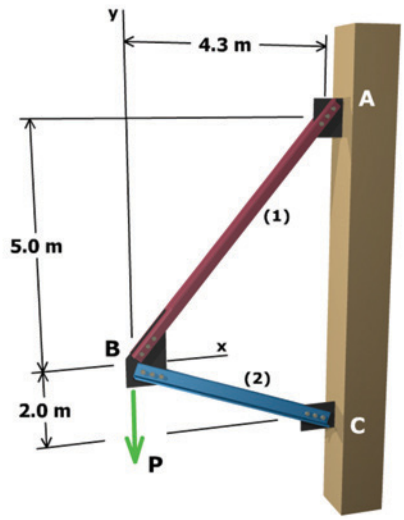
Recheck your calculations, paying particular attention to the units. Always show the units in your calculations because this is an easy and fast way to discover mistakes. Are the answers reasonable? If the bar thickness had been 0.0217 mm instead of 21.7 mm, would this have been a reasonable solution based on your common sense and intuition?

## MecMovies Example M1.4

Two axial members are used to support a load *P* applied at joint *B*.

- Member (1) has a cross-sectional area of $A_1 = 3{,}080$ mm² and an allowable normal stress of 180 MPa.
- Member (2) has a cross-sectional area of $A_2 = 4{,}650$ mm² and an allowable normal stress of 75 MPa.

Determine the maximum load *P* that may be supported without exceeding either allowable normal stress.




# 1.3 Direct Shear Stress

Loads applied to a structure or a machine are generally transmitted to individual members through connections that use rivets, bolts, pins, nails, or welds. In all of these connections, one of the most significant stresses induced is a *shear stress*. In the previous section, normal stress was defined as the intensity of internal force acting on a surface *perpendicular* to the direction of the internal force. Shear stress is also the intensity of internal force, but shear stress acts on a surface that is *parallel* to the internal force.

To investigate shear stress, consider a simple connection in which the force carried by an axial member is transmitted to a support by means of a solid circular pin (Figure 1.2*a*). The load is transmitted from the axial member to the support by **shear force** (i.e., a force that tends to cut) distributed on a transverse cross section of the pin. A free-body diagram

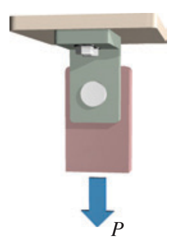



**FIGURE 1.2*a*** Single shear pin connection.

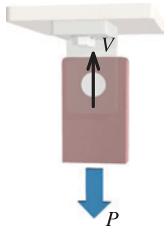



**FIGURE 1.2*b*** Free-body diagram showing shear force transmitted by pin.

**MecMovies 1.7 and 1.8** present animated illustrations of single and double shear bolted connections.

**MecMovies 1.9** presents an animated illustration of a shear key connection between a gear and a shaft.

of the axial member with the pin is shown in Figure 1.2*b*. In this diagram, a resultant shear force $V$ has replaced the distribution of shear force on the transverse cross section of the pin. Equilibrium requires that the resultant shear force $V$ equal the applied load $P$. Since only one cross section of the pin transmits load between the axial member and the support, the pin is said to be in **single shear**.

From the definition of stress given by Equation (1.1), an average shear stress on the transverse cross section of the pin can be computed as

$$\tau_{avg} = \frac{V}{A_V} \tag{1.5}$$

where $A_V$ = area transmitting shear stress. The Greek letter $\tau$ (tau) is commonly used to denote shear stress. A sign convention for shear stress will be presented in a later section of the book.

The stress at a point on the transverse cross section of the pin can be obtained by using the same type of limit process that was used to obtain Equation (1.3) for the normal stress at a point. Thus,

$$\tau = \lim_{\Delta A_V \to 0} \frac{\Delta V}{\Delta A_V} \tag{1.6}$$

It will be shown later in this text that the shear stresses cannot be uniformly distributed over the transverse cross section of a pin or bolt and that the *maximum shear stress* on the transverse cross section may be much larger than the average shear stress obtained by using Equation (1.5). The design of simple connections, however, is usually based on average stress considerations, and this procedure will be followed in this book.

The key to determining shear stress in connections is to visualize the failure surface or surfaces that will be created if the connectors (i.e., pins, bolts, nails, or welds) actually break (i.e., fracture). The shear area $A_V$ that transmits shear force is the area exposed when the connector fractures. Two common types of shear failure surfaces for pinned or bolted connections are shown in Figures 1.3 and 1.4. Laboratory specimens that have failed on a single shear

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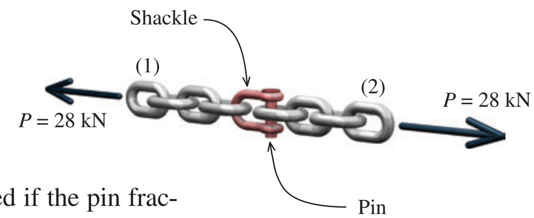
**FIGURE 1.3** Single shear failure in pin specimens.


**FIGURE 1.4** Double shear failure in a pin specimen.

plane are shown in Figure 1.3. Similarly, a pin that has failed on two parallel shear planes is shown in Figure 1.4.

## EXAMPLE 1.4

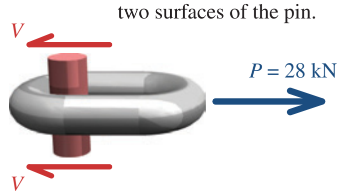
Chain members (1) and (2) are connected by a shackle and pin. If the axial force in the chains is $P = 28$ kN and the allowable shear stress in the pin is $\tau_{allow} = 90$ MPa, determine the minimum acceptable diameter $d$ for the pin.




### Plan the Solution

To solve the problem, first visualize the surfaces that would be revealed if the pin fractured due to the applied load $P$. Shear stress will be developed in the pin on these surfaces, which will occur at the two interfaces (i.e., common boundaries) between the pin and the shackle. The shear area needed to resist the shear force acting on each of these surfaces must be found, and from this area the minimum pin diameter can be calculated.

Shear forces $V$ act on two surfaces of the pin.




**Free-body diagram of pin.**

### SOLUTION

Draw a free-body diagram (FBD) of the pin, which connects chain (2) to the shackle. Two shear forces $V$ will resist the applied load of $P = 28$ kN. The shear force $V$ acting on each surface must equal one-half of the applied load $P$; therefore, $V = 14$ kN.

Next, the area of each surface is simply the cross-sectional area of the pin. The average shear stress acting on each of the pin failure surfaces is, therefore, the shear force $V$ divided by the cross-sectional area of the pin. Since the average shear stress must be limited to 90 MPa, the minimum cross-sectional area required to satisfy the allowable shear stress requirement can be computed as

$$\tau = \frac{V}{A_{pin}} \qquad \therefore A_{pin} \geq \frac{V}{\tau_{allow}} = \frac{(14 \text{ kN})(1{,}000 \text{ N/kN})}{90 \text{ N/mm}^2} = 155.556 \text{ mm}^2$$

The minimum pin diameter required for use in the shackle can be determined from the required cross-sectional area:

$$A_{pin} \geq \frac{\pi}{4} d_{pin}^2 = 155.556 \text{ mm}^2 \qquad \therefore d_{pin} \geq 14.07 \text{ mm} \quad \text{say, } d_{pin} = 15 \text{ mm} \quad \textbf{Ans.}$$

In this connection, two cross sections of the pin are subjected to shear forces $V$; consequently, the pin is said to be in **double shear**.